were observed between measured values of $T_w$ and $q_w$ and the laminar boundary-layer theory for centered expansion waves. If the (unmeasured) waveforms were qualitatively similar to that of Fig. 2 then the observed discrepancies are at least qualitatively explained by the predictions of the present analysis.


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# Hot-Wire Coil Probe for High-Speed Flows


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### Nomenclature

| | |
|---|---|
| $d$ | = diameter of wire |
| $D$ | = diameter of coil |
| $e$ | = compensated hot-wire output voltage |
| $f$ | = frequency |
| $h$ | = wire film coefficient of heat transfer |
| $I$ | = hot-wire heating current |
| $k$ | = thermal conductivity of gas |
| $l$ | = length of wire |
| $M$ | = Mach number |
| $Nu_t$ | = $hd/k_t$ = Nusselt number |
| $Re_{t,d}$ $Re_{t,D}$ | = $(\rho U)_t d/\mu_t$ and $(\rho U)_t D/\mu_t$, respectively, Reynolds number |
| $T$ | = temperature |
| $U$ | = gas velocity |
| $\eta$ | = wire recovery factor ($T_w/T_t$ as $I_w \to 0$) |
| $\mu$ | = gas viscosity |
| $\rho$ | = gas density |

*Subscripts*

| | |
|---|---|
| $\infty$ | = freestream value |
| $d$ | = based on wire diameter |
| $D$ | = based on coil diameter |
| $l$ | = local flow value |
| $w$ | = hot wire |
| $t$ | = local total value |

### Introduction

CONVENTIONAL hot-wire probes for high-speed flows typically have the wire mounted slack between end supports. This slack minimizes wire breakage, and reduces strain gage effects.[1] The wire length to diameter ratio and necessary amount of slack depend on the test environment, and are a compromise between end loss effects and strength. For high wire loading the wire may need to be so short that end conduction losses are large. In addition, the large amount of slack needed can result in wire support interference even when the probe is at a small angle to the mean flow.

One possible solution to some of the limitations of a conventional fine-wire probe is the use of a small diameter coil of wire for the probe. The springlike properties of the coil allow a higher length-to-diameter ratio for a given flow and minimize strain gage effects. In addition, the coil is more rugged for sudden flow changes. Since a coil can be mounted straight across the support tips there is less support interference in cross flows.

This Note describes such "coil probes" developed for use in a hypersonic helium tunnel. In addition to measuring fluctuating quantities in a boundary layer, these probes were used with a constant temperature anemometer for measuring mean mass flow profiles, and with the constant current anemometer for measuring mean total temperature profiles.

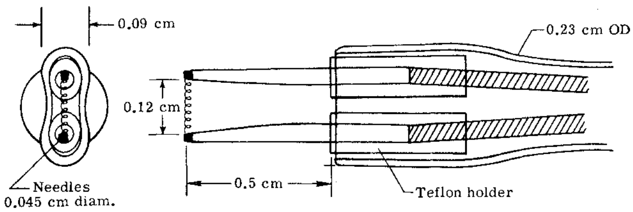


Fig. 1 Hot-wire coil-probe construction.

### Probe Construction and Calibration

A small coil of tungsten wire (from a light bulb) was copper plated, stretched across notched needles, and soft soldered to the needle tips. The copper plating between the needles was then removed with nitric acid. The needles were held in a teflon insulator within a small stainless-steel tube. A sketch of a typical probe is shown in Fig. 1. Different size wires and coils were employed with different $(l/d)$'s, depending on the flow environ-

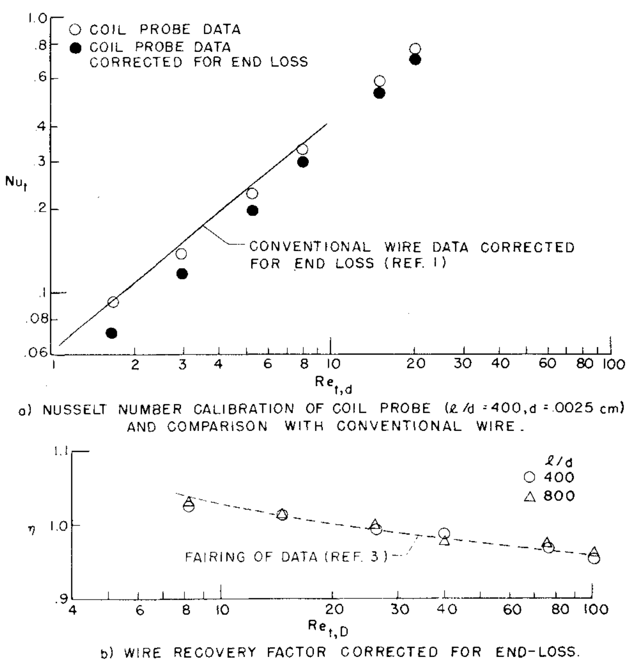


Fig. 2 Typical hot-wire coil-probe calibrations.


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ment. Typical probes had wire diameters from 0.001 to 0.0025 cm, coil diameters from 0.010 to 0.013 cm, and wire $(l/d)$'s from 400 to 1500.

Probes were calibrated in two facilities using helium as a test medium. The first facility was a 7.6-cm exit diameter conical nozzle which had a stream Mach number near 20. The second facility was a low-density nozzle which had a stream Mach number near 4. A description of the flow conditions of these nozzles can be found in Ref. 2. The probe calibration and data reduction procedures were essentially identical to those of Ref. 1.

Typical calibrations for $Nu_t$ against $Re_{t,d}$ and $\eta$ against $Re_{t,D}$ are shown in Fig. 2a and 2b, respectively. In Fig. 2a, $Nu_t$ and $Re_{t,d}$ were based on wire dimensions, since the heat transfer took place on the wire surface. Data from a coiled wire are given by the open symbols. The same data with calculated end losses removed are shown by solid symbols. The slope of the corrected data is 0.91, and is typical for the coil probes examined. A calibration curve for a conventional probe from Ref. 1 was corrected for end losses and is shown for comparison. The conventional probe calibration curve has a similar level but a smaller slope than that for the coiled probe. This may be due to the different local flows around the two probes. The coil probe recovery factor corrected for end loss is shown in Fig. 2b for two different $(l/d)$'s. The recovery factor is shown against $Re_{t,D}$ along with a fairing of recovery factor data for a straight cylinder with end loss effects removed.[3] The value of $\eta$ for both $(l/d)$'s agrees with the data of Ref. 3 indicating that the probe shock was probably detached ahead of the entire coil rather than each wire loop. The fairing from the data of Ref. 3 is given here by the dashed line for comparison.

### Experimental Studies Using Coil Probes

The nozzle wall boundary layer in the 7.6 cm, $M \simeq 20$ facility is turbulent at higher operating pressures[4] and radiates sound into the freestream flow.[5] When a new design probe is tested in this nozzle, the fluctuation spectrum obtained can be compared with that of conventional hot wires to determine response characteristics. The freestream disturbance spectrum obtained with a long wire ($l/d = 1500$) coil probe in this facility is shown in Fig. 3. The spectrum from a conventional wire[4] is also shown in Fig. 3 for comparison. The spectrum was the same as that of the conventional wire except for a small spike at $\simeq 5$ KHz. The spike was evidently caused by a resonant vibration of the coil at this frequency. The effect of this spike is generally small and can be neglected for either mean flow measurements or wide band fluctuation measurements. However, for some cases, this vibration may have to be considered (e.g., narrow band measurements near the resonant frequency).

To further check the probe accuracy, mean boundary-layer measurements were obtained on a cone in the Langley $M_\infty \simeq 20$ High Reynolds number helium tunnel using the coil probes. Details of the facility can be found in Ref. 6. The boundary layer on the cone (cone half angle = 2.87° cone at $3\frac{1}{2}°$ angle of attack) was probed with both a pitot probe and a coil probe to determine 3-D transitional and turbulent boundary-layer profiles at a local Mach number of $\simeq 10$. Since the model wall temperature was near the flow total temperature, the pitot data was reduced by assuming constant total temperature. The hot-wire temperature survey verified that the total temperature variation through the boundary layer was less than 5%. Typically

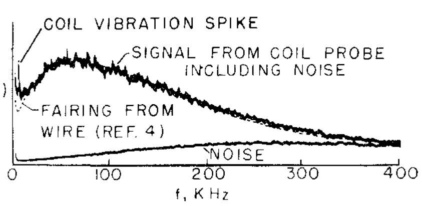


**Fig. 3 Spectrum from hot-wire coil-probe in 7.6-cm helium nozzle.**

mass flow profiles obtained with both pitot probe and coil hot-wire probe showed agreement to within 4% throughout the entire boundary layer.

In concluding, the coil hot-wire probe exhibits all the advantages of conventional hot-wire probes, and few of the disadvantages. Thus the coil probe is particularly useful for high-speed flow studies where probe breakage and large end loss errors are bothersome.

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# Design of Least Weight Structures for Prescribed Buckling Load

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### 1. Introduction

DESPITE the significant effort of the past decade directed toward advancing analytical techniques for optimal design of single-span columns, optimization of complex structures under prescribed buckling load using numerical methods has apparently not been reported. However, using various discretization procedures, considerable attention has been paid to the nearly equivalent mathematical problem involving the design of structures subject to a frequency constraint. Of these procedures, the gradient method of Zarghamee[1] and Rubin[2] seems to be well suited for extension to buckling problems. Whereas the method originally comprised a finite element formulation in which only a linear weight-stiffness relationship was considered, here a quadratic relationship is used. Further, an averaging technique is described which permits the design parameter to vary uniformly over preassigned segments. The optimal design of a two-span continuous column is illustrated.

### 2. Optimization Method

The procedure begins with an initial design which is modified so as to satisfy the buckling load constraint; then, holding the buckling load constant, the weight is minimized.

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